# Informal firms, investment incentives and formalization

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## Abstract

In a typical developing country, the majority of small firms are informal and entry costs into formality are high. This paper is motivated by these two observations. It asks the question of what can be expected in terms of firm investment, growth and formalization in such a setting. It also studies the effects of policies towards the informal sector on formalization decisions. I show that the investment paths and growth trajectories differ substantially between firms that choose to formalize and those, with similar productivity levels, that do not. Second, the formalization decision depends non-trivially on the productivity of the informal firm, due to the balancing of an accumulation effect and a threshold effect. This, in turn, has an effect on how policies towards the informal sector should be designed. Third, when aggregating over firms, the long-run firm size distribution exhibits a range of small firms and a range of larger firms, but also a "missing middle", much in line with actual firm size distributions observed in developing countries. Fourth, the long-run firm-size distribution turns out to depend on the initial firm-level stock of capital, a result that resembles a poverty/informality trap.

**Key words:** Informal firms, Investment, Entry costs, Non-convexities, Formalization

## 1. Introduction

In a typical developing country, the majority of small firms are informal and entry costs into formality are high. This paper is motivated by these two observations. It addresses the question of what can be expected in terms of firm investment, growth and formalization in such a setting. In particular, the paper focuses on firms' incentive to invest when, at some future point in time, an increase in productivity can be gained, but only after paying large entry costs into formality. The effect of a penalty policy on informal firm investment, growth and formalization is also discussed.

The observation that most small firms in developing countries are informal is well-established. A recent enterprise survey in Brazil shows that 90% of the smallest firms, i.e. firms with 1-5 employees, have not gone through the procedure to register as a legal entity (SEBRAE, 2005). An enterprise survey in Mexico, the other large Latin American economy, shows similar values (INEGI, 2003). Studies and accounts from other developing countries indicate similar degrees of informality among the smallest firms in the economy (see, for instance, Bigsten et al., 2004, for Kenya and de Soto, 1989, for Peru).

Turning to bureaucratic and legal costs facing small and medium enterprises, such costs have received considerable attention in recent development research. In particular, the work by de Soto (1989) and Djankov et al. (2002) has directed the attention to substantial government-related costs of "doing business" and entry into formality. Examples of such costs are start-up fees, financial costs incurred in order to pay taxes (except for the taxes themselves), financial costs related to hiring and laying off workers, as well as the time spent with these activities. These costs can be substantial. Whereas it costs USD 370 to start a firm in the US, the average cost in Latin America is around USD 1240, as reported by the World Bank Doing Business project. The average monthly income per capita was USD 3840 in the United States in 2007, meaning that three days of work generate an income equal to the firm start-up cost. In Latin America, the average monthly income was one tenth as much, or USD 380. It thus takes more than three months of work to generate an income equal to the firm start-up cost. Furthermore, income levels in the informal sector in Latin America are typically much lower than the official GNI figures, meaning that it takes even longer to generate an income equal to the firm start up costs.

As implied by the above, the definition of an informal firm used in this paper is a firm that has not gone through the registration procedure at the government bureaucracy.

The combination of small informal firms and large formalization costs has motivated setting up a simple dynamic model of profit-maximizing firms. Firms can invest in their capital stock, grow larger and, possibly over time, become formal. The cost of becoming formal is taken literally: at one instant in time, the firm can choose to pay the formalization fee, defined as $F$**;** a fee that represents all costs to register the firm at the government bureaucracy. Having paid $F$, the firm changes status from informal to formal and obtains a productivity benefit.

How do formalization costs, to be paid at some future date, affect investment today? At what firm size and when do firms choose to become formal, if at all? What are the crucial parameters affecting firm formalization? What is the effect of credit constraints on the formalization decision? Can formalization costs lead to poverty traps? How should policy vis-a-vis informal firms be viewed? How can the government affect the formalization decision? These questions are addressed in this paper.

Several interesting results emerge from the analysis of the tractable dynamic model. First, the investment paths and growth trajectories differ substantially between firms that choose to formalize and those (ex-ante almost identical firms) that do not. Second, the formalization decision depends non-trivially on the productivity of the informal firm, due to the balancing of an *accumulation effect* and a *threshold effect*. This, in turn, has an effect on how policy designed to incentivize informal firms to become formal should be designed. Third, when aggregating over firms, the long-run firm size distribution exhibits a range of small firms and a range of larger firms but also a "missing middle", much in line with actual firm size distributions observed in developing countries (Bigsten et al. 2004, Tybout, 2000). Fourth, the long-run firm-size distribution turns out to depend on the initial firm-level stock of capital, a result that can be interpreted as a poverty/informality trap.

The paper proceeds as follows: In section 2, the literature to which this paper relates is reviewed and the model to be presented is motivated in relation to this earlier writing. Section 3 discusses formalization costs in different countries and presents some data on income levels in the informal economy, together with typical informal firm capital stocks and profit levels from three recent studies. The dynamic model of firm investment and formalization is presented in section 4 and analyzed in section 5. Some extensions to the analysis, focusing on how the investment and formalization behavior changes when the basic assumptions are altered, are to be found in section 6. Section 7 discusses the results and concludes the paper. The appendix presents some of the details in deriving the analytical results.

## 2. Literature review

An important debate in the literature on the informal sector, preceding the analysis in this paper, is whether small informal entrepreneurial activities should be considered as proper "firms" at all, or merely as temporary subsistence labor while waiting for a formal job. In early writings on how the economy develops from traditional to modern, Lewis (1954), Todaro (1969) and Harris and Todaro (1971) considered the "urban traditional" sector as a source of labor supply for the "modern" sector.[1] In none of these papers is the urban traditional sector seen as an important element of economic activity or as a contributor to capital accumulation. It is rather considered as a temporary low-productivity subsistence activity.

The entrepreneurial view that informal small-scale economic activities should be considered as entrepreneurs/firms, rather than as subsistence activities, has been popularized by de Soto (1989). However, a change in terminology and focus to an informal – rather than an "urban traditional" – sector, stressing entrepreneurship and not only surplus labor, emerged with the writings of the International Labor Organization (Hart, 1973; ILO, 1972). The informal sector/informal economy started to be seen more as a permanent, increasing and diverse phenomenon – "from marginal operations to large enterprises" (Hart, p. 68).[2]

In an "occupational choice" model in the spirit of Lucas (1978), with economic agents differing in entrepreneurial ability, Rauch (1991) studies the choice between being a worker, an informal entrepreneur or a formal entrepreneur. Both types of entrepreneurs employ workers. The informal entrepreneurial sector arises as a result of a government (above-market clearing) minimum wage policy. The static general equilibrium model delivers predictions on the relative size of the informal sector, firm size distribution, and changes to these from the minimum wage level.[3]

Minimum wages that do not clear the market constitute an example of a government intervention that may lead to an informal sector. The focus here is instead on the effects of government-imposed formalization costs. The paper takes as given the de Soto entrepreneurial view and studies investment and formalization decisions of profit maximizing informal firms.

The question of whether a firm formalizes or not in the face of large such costs involves at least two issues: the formalization costs themselves and the potential gains from formalization. In addition, a modeling choice must be made. A dynamic framework is appropriate to capture the effects of large formalization costs on small informal firms: firms must grow to a certain size to become formal. A dynamic model can also shed light on how the investment incentives and the resulting growth path today are affected by a future "non-convexity" in the production function.

To the best of my knowledge, this paper is the first to explicitly focus on the investment incentives in anticipation of a formalization cost. However, the model is similar in spirit to the literature on non-convexities and poverty traps, a literature that typically focuses on whether initial (wealth) conditions matter for long-run allocations.[4]

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[1] Starting with the work of Lewis (1954), the traditional urban sector plus rural-to-urban migrants were seen as a source of unlimited labor supply from which the modern sector could get labor at subsistence pay. Todaro (1969) modeled the rural–to urban migration decision, taking into account the existence of an unemployed or underemployed pool of urban traditional workers that compete for the same jobs as rural migrants. Harris and Todaro (1971) studied a minimum wage policy in a similar setting.

[2] Rauch (1991), Chen (2004) and de Mel et al. (2008a) all discuss early writings on the informal economy.

[3] Rauch's paper can be seen as combining the two views above on informal activity. In recent empirical work from Sri Lanka, de Mel et al. (2008a) collect data on personal characteristics from wage workers, own-account workers and owners of enterprises with 5-50 employees to address the question of whether own-account workers resemble wage workers or firm-owners more. By using a "species classification" approach from biology, they classify around 70% of the own-account workers in their study as wage workers and 30% as small- and medium-size enterprise owners. On the other hand, the authors argue that given the large number of own-account workers in low-income countries, the possibilities for job creation and growth in this sector should not be ignored. In addition, by exploiting the panel structure of the data on own-account workers, for the fraction of this group that resembles enterprise owners, the growth rate implied by these own-account workers' transition into being employers is much larger than the growth rates found in a comparable study with data from the United States.

[4] Regarding terminology, the present paper discusses government imposed formalization costs, in the form of going through a firm registration procedure, as the fundamental non-convexity which is of importance for firm growth. This is different from occupational choice models, such as Banerjee and Newman (1993), Ghatak and Jiang (2002) and Buera (2008), where the non-convexity is typically a minimum scale investment. The two different types of entry costs may well operate on different levels of firm size: an individual considering starting a manufacturing "firm" may consider buying a machine ("entry"). After having grown, such an informal manufacturing firm, with an established operation and possibly with a number of employees, may consider "formalization". In a recent empirical paper on the return to capital of investment

One basic insight from neoclassical theory is that non-convexities alone will not affect long-run allocations. Economic agents could simply borrow to overcome such hurdles. The analysis of models with non-convexities is therefore intimately connected with introducing some other constraint relevant for developing economies, in particular capital market imperfections which may make individuals or firms unable to converge to a common long-run steady state or a balanced growth path (Banerjee, 2001 and McKenzie and Woodruff, 2006 discuss this point). The effects of initial capital and credit constraints on the possibilities for firm formalization are discussed in this paper.

Typically, the interplay between non-convexities, credit constraints and initial wealth is studied in dynamic occupational choice models with an OLG-structure, where one generation bequests wealth to the next and where individuals have a "warm glow" utility function. In the baseline human capital investment model of Galor and Zeira (1993), this results in a direct relationship between the initial wealth of one generation of a dynasty and the long-run steady state of the same dynasty.[5] There is no intergenerational saving/investment where a current generation takes into consideration the possibility that the decision of a future generation may be affected by today's choice. The framework in this paper is different. Firms maximize profits over the entire life span of the firm. This means that the investment decision is truly intertemporal. The firm considers whether it should build up a capital stock over time, although this may imply current losses, in order to formalize at some later point in time.[6]

A feature of the present model, as opposed to most other papers, is that it is possible to solve analytically for the shape of the investment function over time. The comparative statics of the model can thus be analyzed in a straightforward way.

Turning to the second issue, what is to be gained from formalization? This paper assumes that there is a productivity gain from becoming formal and focuses on the resulting effect on investment incentives while informal, but it does not provide one specific channel through which formal productivity is higher.

A non-exhaustive list of aspects that differ between informal and formal firms, from the development literature, includes access to credit and capital, taxes, public goods provisioning, access to risk pooling mechanisms, security in business environment, property rights, marketing possibilities, access to export markets, supplier-buyer relationships and other contracting issues (see, for instance, de Soto, 1989; Tokman, 1992; Levenson and Maloney, 1998; Bigsten et al. 2004; Chen, 2004; Maloney, 2004).[7,8]

One mechanism, out of many possible, that affects the productivity of informal firms and, therefore, the incentive to become formal, is instead proposed: Penalties and enforcement vis-a-vis informal firms make these firms divert time from production, with lower total production as a result. Tokman (1992) provides ample evidence that informal firms in Latin America organize part of production so that it is "invisible". The accounts in Tokman contain numerous examples of how small informal firms organize activities to minimize the disturbance from authorities, for instance by choosing less visible and less favorable production locations, physically hiding production when authorities visit and in anticipation of such visits, meeting customers one by one due to the lack of a visible sales location and marketing possibilities, and so on. The set-up, where firms respond to penalties by diverting time from production, allows us to explicitly study the effect of changes in policy, i.e. penalties, on informal firm investments and decisions to formalize.

The main focus of the paper is to study the investment incentives of an individual firm. However, the aggregate formalization behavior of heterogenous firms – differing in an ability parameter (or in initial capital) – is also studied. The aim is not to provide an industry evolution model, as in Jovanovic (1982),

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for small firms in Mexico, McKenzie and Woodruff (2006) find, in line with other papers, high returns on small investments for the smallest firms and thus, they find no evidence of "entry nonconvexities". They do find lower returns for firms with a capital stock in the USD 1000-2000 range, however, and cannot reject that there is a threshold effect, one potential explanation for which is that "fiscal and bureaucratic costs are faced only by firms above a minimum size" (McKenzie and Woodruff, 2006, page 5).

[5] In an extension, as well as in the occupational choice models of Banerjee and Newman (1993) and Ghatak and Jiang (2002), the entire wealth distribution endogenously determines occupational choices and wages which, in turn, affect the bequest to the next generation and the long-run equilibrium.

[6] See Banerjee (2001 pp. 31-32) for a discussion of "joy of giving" vs. "Barro preferences". In an appendix, Galor and Zeira (1993) point out how a "utility of offspring"-approach would affect their results and show that a poverty trap would still result. The occupational choice model of Buera (2008) also uses fully intertemporal preferences.

[7] The effects of taxation in the formal sector and of differences in public goods provisioning between sectors are modeled by Loayza (1995) and Garcia Penalosa and Turnovsky (2005). Differences in access to outside finance are modeled by Antunes and Cavalcanti (2007).

[8] The assumption that formality brings a productivity benefit is not uncontroversial. As an example, much of the discussion in Brazil is centered around (too high) taxation in the formal sector. This paper assumes that formality is desirable, although the framework could, in principle, allow for firms that do not desire formality.

Hopenhayn (1992) and Melitz (2003). It is rather to display the implications of the non-convexity on long-run firm sizes and formality status, when firms differ in ability and initial capital. These "aggregate" predictions of the model are outlined in section 5.

### 3. Formalization costs

The cost of formalizing a business consists of both monetary costs and other costs. It is well documented that these costs can be very high (Djankov et al., 2002). The most up-to date source of information on such costs is most likely the "Doing Business" project financed by the World Bank. This data set on costs to start a firm originally covered 75 countries (Djankov et al., 2002), while 181 countries are now included (World Bank, 2009a). A summary of the most recent data, from 2009, is presented in table 1, with the number of procedures to register a firm and the official time it takes. The last column measures the official cost of the different registration procedures as a percentage of official Gross National Income (GNI). The financial cost to start a business (column 3) is at least 30% of yearly GNI per capita in most of the developing world, and as much as 111% in Sub-Saharan Africa.

**Table 1 about here**.

Table 2 presents data for the year 2007 for the Latin American countries present in the World Bank data, augmented with informal economy income figures from Schneider (2002). Columns 1-3 show that 6-17 different bureaucratic procedures with a total cost of 585-2820 USD and taking 19-152 days have to be taken to formalize a firm. The average is 12 procedures, 58 days and 1238 USD in official cost. All Latin American countries have a higher firm start-up cost than the United States and the average cost is 336% of the US cost.[9] Column 4 shows the official 2007 GNI/capita figures from World Bank (2009b), column 5 shows the informal GNP/capita figures from Schneider (2002) and columns 6-7 show the ratio between the cost to start a firm to the monthly informal GNP/capita and the ratio between a "total cost" to the monthly informal GNP/capita, respectively.[10,11] Columns 6 and 7 can thus be interpreted as the number of months an average informal worker would have to work to generate an income equal to the official firm start-up cost and the total cost, respectively. If we only focus on the official cost to start a firm (column 6), then Brazil, the most favorable country, requires three months of work to generate the income required for the formalization cost. The Latin-American average is 11 times informal GNP/capita and Bolivia and Nicaragua have very high costs in terms of informal income. These costs are high and are likely to be prohibitive for many small informal firms.

**Table 2 about here**.

To finish this section, three examples on capital stock levels and profits from small (typically informal) firms are given. In a representative sample of 3700 firms with five employees or less in Mexico, McKenzie and Woodruff (2006) report that the median capital stock replacement value across industries is USD 963. In Mexico, typical capital stocks are thus worth less than the costs of going through the registration procedure, from table 2. In the same Mexican data set, the average reported monthly earnings for firms with less than the median capital stock are USD 172 (Woodruff, 2006). In another study, from Sri Lanka, De Mel, McKenzie and Woodruff (2008b) report that the median level of invested capital for 408 firms is around USD 180.[12]

[9] All averages calculated are unweighted.

[10] To get an informal economy per capita income relevant for 2007, I have multiplied Schneider's informal economy per capita GNP figures, which refer to the year 2000, with the ratio between 2007 and 2000 official income figures. The calculation thus assumes that the informal economy per capita income has changed at the same rate as the official per capita income.

[11] The total cost measure, as perceived by an informal entrepreneur, is probably a summary measure of the monetary cost + the time cost of actually fulfilling all requirements + transport costs etc. to visit the different government bodies. The calculation for total cost in column 7 is somewhat ad-hoc and, as follows: (the official cost) + (the number of procedures times half the informal average daily GNP/capita) + (an ad-hoc measure of the loss of waiting set to the duration in days divided by three times half the daily informal GNP/capita). The daily GNP/capita is the monthly GNP/capita divided by 20. Each procedure is assumed to require one day of work. Each procedure is assumed to have a value of half an average daily informal GNP/capita. The loss due to waiting is set to be a third of the duration time times half the daily GNP/capita.

[12] In the latter study, firms with less than 1000 USD in capital stock were targeted, which caused around 6% of the originally selected sample of entrepreneurs to be dropped. The authors argue that "we believe the resulting sample is representative of a substantial majority of the own-account workers in Sri Lanka" (page 1335, footnote 4).

As an example of informal firm profits, the Brazilian study of informal 1-5 person firms cited in the introduction reports that roughly 75% of the firms say that they make profits. The average monthly profit of these profit-making firms was USD 314. The profit for firms with remunerated employees was USD 825. For own account firms/workers, that may or may not have non-remunerated employees, the profits were USD 235 (SEBRAE, 2005). These entrepreneurial activities are often the main or the sole activity of the individuals involved, indicating a small room for anything but consumption expenses.[13]

## 4. The model

In this section, a dynamic model of firm investment, growth and possible formalization is introduced and solved. The firm starts out as informal and the question is if, when and at what firm size the firm will become formal. The modeling is inspired by the framework in Harstad and Svensson (2009).

The production function is simple: production is linear in the capital stock ($k_t$). As informal, the firm produces $A^i k_t$, if it has formalized, production is instead $A^f k_t$, where $A^f > A^i$. Thus, it is assumed that formality is desirable for the firm. I first solve a dynamic profit maximization problem in sections 4.1 to 4.5. Because the focus in section 4 is on one individual firm, heterogeneity between firms is not introduced until section 4.6, after which I also discuss a possible microfoundation for $A^i$.

The firm can grow by investing ($i_t$) in its capital stock. The cost of investing is convex in the size of the investment, $\frac{z}{2}i_t^2$. This gives a profit flow ($\pi_t$) in the case of the firm being informal, as follows:

$$\pi_t = A^i k_t - \frac{z}{2} i_t^2 \tag{1}$$

The capital stock depreciates at the rate $\delta$. The growth of the capital stock is therefore

$$\dot{k}_t = i_t - \delta k_t. \tag{2}$$

To get access to the higher productivity, $A^f$, the firm must pay a formalization fee $F$ at some time $T$. After formalization, flow profits equal $A^f k_t - \frac{z}{2} i_t^2$. The firm discounts future profits at the rate $\rho$.

The basic dynamic problem, in an environment with no restrictions on how the firm can finance investment and formalization costs from its own lifetime revenue, is stated below. The effect of credit constraints turn out to affect the dynamic analysis in a way which can be handled within the main framework, and is postponed to section 6.

### *4.1. The firm profit maximization problem*

An informal firm, starting with an initial capital stock of $k_0$, chooses an investment path, whether it should become formal and the time of formalization ($T$). The profit maximization problem is:

$$\begin{aligned} &\text{Choose } i_t,\ T \text{ to Max} \left[ \int_0^T \left( A^i k_t - \frac{z}{2} i_t^2 \right) e^{-\rho t} dt + \int_T^\infty \left( A^f k_t - \frac{z}{2} i_t^2 \right) e^{-\rho t} dt - F e^{-\rho T} \right] \\ &\text{s.t. } \dot{k}_t = i_t - \delta k_t \text{ and } k(0) = k_0 \end{aligned} \tag{3}$$

The problem is solved in two steps. First, we use the principle of optimality to solve backwards for the formal and then the informal investment path (assuming that $T$ exists). We also derive the investment path if $T$ does not exist. By using the investment path assuming that formalization does take place, we then determine when the firm wants to formalize by solving for the optimal $T$. If such a $T$ exists, we then know the optimal capital accumulation path. If it does not exist, the firm is informal forever.

[13] An average exchange rate of 2.86 Reais/USD in October 2003 was used to calculate these numbers.

*4.2. Optimal investments*

Assume that $T$ exists. Solving backwards, the "formal problem" takes the capital stock at time $T$, defined as $\tilde{k}_T$, as an initial condition, and is solved for the investment path from $T$ to $\infty$. We get a formal investment function $i^{\text{formal}}$ and a continuation value $V^{\text{formal}}$, which is the optimal profit from $T$ and onwards. $V^{\text{formal}}$ will be a function of both $T$ and $\tilde{k}_T$. The profit maximization problem is:[14]

$$\text{Choose } i_t \text{ to Max} \int_T^{\infty} \left(A^f k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt \qquad \text{s.t.} \qquad \dot{k}_t = i_t - \delta k_t \text{ and } k(T) = \tilde{k}_T$$

By defining the present-value Hamiltonian $H = \left(A^f k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} + \lambda_t (i_t - \delta k_t)$, where $\lambda_t$ is the present value Lagrange multiplier on the capital accumulation constraint, and applying the first-order conditions $\frac{\partial H}{\partial i} = 0$, $\frac{\partial H}{\partial k} = -\frac{\partial \lambda}{\partial t}$ and the transversality condition $\text{Lim}_{t\to\infty}(\lambda_t k_t) = 0$, we get:

$$i^{\text{formal}} = \frac{A^f}{z(\delta+\rho)} \qquad k_t^{\text{formal}} = \tilde{k}_T e^{-\delta(t-T)} + \frac{A^f}{z\delta(\delta+\rho)}\left(1 - e^{-\delta(t-T)}\right) \qquad k_{\infty}^{\text{formal}} = \frac{A^f}{z\delta(\delta+\rho)}$$
$$V^{\text{formal}}\left(T, \tilde{k}_T\right) = e^{-\rho T}\left(\frac{A^f \tilde{k}_T}{\delta+\rho} + \frac{\left(A^f\right)^2}{2z\rho(\delta+\rho)^2}\right) \tag{4}$$

The firm invests a constant amount each "period". The capital stock converges to its steady state value of $k_{\infty}^{\text{formal}} = \frac{A^f}{z\delta(\delta+\rho)}$, at which depreciation and investment offset each other.[15] The constant investment rate is due to the convexity of investment costs – the firm wants to spread investment over time. The investment rate increases in the productivity parameter $A^f$ and decreases in the cost of investment $z$, the depreciation rate of capital $\delta$ and the rate of time preference $\rho$.

The informal investment path, for a given $T$, can, in turn, be determined by solving for the investment path that takes the firm from $k_0$ to $\tilde{k}_T$ and then maximize total profits with respect to $\tilde{k}_T$:

$$\text{Choose } i_t \text{ and } \tilde{k}_T \text{ to Max}\left[\int_0^T \left(A^i k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt + e^{-\rho T} V^{\text{formal}}\left(T, \tilde{k}_T\right)\right]$$
$$\text{s.t. } \dot{k}_t = i_t - \delta k_t,\ k(0) = k_0 \text{ and } k(T) = \tilde{k}_T \tag{5}$$

The investment path is derived as above, the only difference being the terminal constraint on capital (instead of a transversality condition). Having solved for the optimal informal $i_t$- and $k_t$-paths as functions of $\tilde{k}_T$, and having plugged these back into the profit function, we integrate to get the optimal value of informal profits as a function of $\tilde{k}_T$. The total profits are then differentiated with respect to $\tilde{k}_T$. The optimality condition with respect to $\tilde{k}_T$, stated below, is that the loss of informal profits from increasing $\tilde{k}_T$ should be exactly offset by a gain in formal profits:

$$\frac{d}{d\tilde{k}_T}\left(\int_0^T \left(A^i k_t\left(\tilde{k}_T\right) - \frac{z}{2}\left(i_t\left(\tilde{k}_T\right)\right)^2\right) e^{-\rho t} dt + e^{-\rho T} V^{\text{formal}}\left(T, \tilde{k}_T\right)\right) = 0 \tag{6}$$

This equation is solved for $\tilde{k}_T$ and plugged back into the solution for $i_t$ and $k_t$, which become

$$i_t^{\text{formalization}} = \frac{A^i}{z(\delta+\rho)} + \frac{A^f - A^i}{z(\delta+\rho)} e^{(\delta+\rho)(t-T)} \tag{7}$$
$$k_t^{\text{formalization}} = k_0 e^{-\delta t} + \frac{A^i\left(1 - e^{-\delta t}\right)}{z\delta(\delta+\rho)} + \frac{\left(A^f - A^i\right)\left(e^{(\delta+\rho)(t-T)} - e^{-(\delta+\rho)T-\delta t}\right)}{z(\delta+\rho)(2\delta+\rho)}.$$

[14] $\tilde{k}_T$ is not a choice variable in the overall problem, it is only introduced as an auxiliary variable when we solve the formal and informal problems separately.

[15] In solving the problem, a non-explosive path of investment is profit-maximizing. Other investment paths, that fulfill the differential equations for $i_t$ and $k_t$ stemming from the first- order conditions on the Hamiltonian, can be ruled out for optimality reasons (and do not fulfill $\text{Lim}_{t\to\infty}(\lambda_t k_t) = 0$).

This investment path starts out close to $A^i/(z(\delta+\rho))$, and then increases up to the level of formal investments at $T$, i.e. $A^f/(z(\delta+\rho))$. Investment increases close to formalization because the marginal value of capital is high after formalization, which makes the firm willing to decrease its profits by accumulating more capital, while still being informal.

Now assume that $T$ does not exist. The firm is then informal forever. Solving this problem is identical to solving the formality problem above, but productivity is $A^i$, time runs from 0 and the initial capital stock is $k_0$. The "ever-informal" problem is:

$$\text{Choose } i_t \text{ to Max} \int_0^\infty \left(A^i k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt \qquad \text{s.t.} \qquad \dot{k}_t = i_t - \delta k_t \text{ and } k(0) = k_0$$

The solution, obtained as in the formal problem above, is:

$$i^{\text{informal}} = \frac{A^i}{z(\delta+\rho)} \qquad k_t^{\text{informal}} = k_0 e^{-\delta t} + \frac{A^i}{z\delta(\delta+\rho)}\left(1 - e^{-\delta t}\right) \qquad k_\infty^{\text{informal}} = \frac{A^i}{z\delta(\delta+\rho)} \tag{8}$$

The investment rate is constant and the capital stock converges to a steady-state value, $k_\infty^{\text{informal}} = \frac{A^i}{z\delta(\delta+\rho)}$. This capital stock is lower than if the firm had been formal, because productivity is lower.

*4.3. Solving for the formalization time $T$*

If $T$ exists, the investment path before and after formalization is given above (expressions 7 and 4, respectively). The optimal $T$ can be derived by recognizing that at the time of formalization, it must be that formalization is just as attractive as remaining informal. This determines the capital stock at which the firm wants to formalize which, in turn, with the capital accumulation prior to formalization $k_t^{\text{formalization}}$ given in (7), determines $T$. We get that formalization takes place when

$$\frac{d}{dT}\left(\int_0^T \left(A^i k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt + \int_T^\infty \left(A^f k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt - F e^{-\rho T}\right) = 0. \tag{9}$$

As discussed above, the pre-formalization investment rate approaches the formal investment rate as $t \to T$. At $T$, these effects cancel out and the condition in (9) simplifies to $A^i k_T - A^f k_T + \rho F = 0$. The optimal capital stock at formalization, defined as $k^F$, becomes

$$k^F \equiv \frac{\rho F}{A^f - A^i}. \tag{10}$$

We get $T$ by equating the capital accumulation path at $t = T$, i.e. $k_T^{\text{formalization}}$ from (7), with $k^F$:

$$k_0 e^{-\delta T} + \frac{A^i\left(1 - e^{-\delta T}\right)}{z\delta(\delta+\rho)} + \frac{\left(A^f - A^i\right)\left(1 - e^{-(2\delta+\rho)T}\right)}{z(\delta+\rho)(2\delta+\rho)} = \frac{\rho F}{A^f - A^i} \tag{11}$$

This equation implicitly defines the optimal time of formalization, $T$.

Formalization means a promise of future higher profits. The firm that formalizes builds a higher capital stock while informal, in anticipation of such profits. Because $i_t^{\text{formalization}} > i^{\text{informal}}$, this period is thus associated with losses as compared to the "ever-informal" path. There is a certain amount of losses/additional investment that can be sustained in anticipation of formalization. This gets reflected in the amount of capital that is optimally accumulated prior to formalization, i.e. the LHS in (11).

The formalization decision also depends on at what capital stock it is optimal to pay $F$. The first-order condition in (9) implies that the marginal gain from formalization, which is $\left(A^f - A^i\right)$ times the capital stock, should equal the marginal loss of not delaying formalization, i.e. $\rho F$.

It should be observed at this stage that although we have not restricted the time of payment of $F$ in any sense, the firm *does not* want to pay the formalization fee at once. This is because it is only beneficial to pay $F$ once a certain capital stock/firm size has been reached and getting to that point is costly due to the convexity of investment costs.

*4.3.1. Existence of T*

Determining under what conditions $T$ exists completes the solution to the dynamic problem. Proposition 1 below states the full conditions for when a firm formalizes. The main idea in deriving this proposition is to let $T \to \infty$ in the LHS of expression (11), which gives an auxiliary maximum level of capital $k_{\infty}^{\text{formalization}} = \dfrac{A^i(\delta+\rho)+\delta A^f}{z\delta(\delta+\rho)(2\delta+\rho)}$ in anticipation of formalization and then to compare this capital level to the RHS in (11). Appendix 1 gives some further details.

**Proposition 1**: A firm that starts with a capital level $k_0$ less than $k_{\infty}^{\text{informal}} = \dfrac{A^i}{z\delta(\delta+\rho)}$ will become formal if and only if the formalization cost $F$ is less than or equal to $\bar{F}$, where $\bar{F} \equiv \dfrac{\left(A^f - A^i\right)\left(A^i(\delta+\rho)+\delta A^f\right)}{z\delta\rho(\delta+\rho)(2\delta+\rho)}$. This threshold is increasing in $A^f$, decreasing in $z$, $\delta$ and $\rho$ and increasing in $A^i$ for small values of $A^i$, and then decreasing. For firms that start with $k_0$ larger than $k_{\infty}^{\text{informal}}$, formalization will take place if and only if $F \leq \bar{F} + G(k_0)$, where $G(k_0)$ is positive and a strictly increasing function of $k_0$.

The next subsection states the full solution. A second proposition is then presented, after which the basic comparative statics and the intuition of firm formalization are discussed. The discussion of the second part of proposition 1, the $k_0$-dependence, is postponed until section 5.

*4.4. The full solution to the dynamic problem*

The solution to the dynamic problem can be stated as follows: If the conditions in proposition 1 are satisfied, there exists a formalization time $T$ which is the solution to equation 11. In this case, the firm follows the formalization investment path ($i_t^{\text{formalization}}$ from expression 7) and then switches to the formal investment path ($i^{\text{formal}}$ from expression 4) at time $T$. Such an investment path is shown in figure 1. If instead proposition 1 is not satisfied, the firm follows an "informal-ever" investment path ($i^{\text{informal}}$ from expression 8, the broken line in figure 1).

**Figure 1 about here.**

*4.5. Comparative statics of the dynamic problem*

Proposition 1 was derived from expression (11). An alternative approach to the above is to use expression (11) to analyze the comparative statics of the time of formalization, $T$. The same parameter changes that make formalization "easier" (reflected in an increase in $\bar{F} - F$) also imply a smaller $T$.

**Proposition 2**: The formalization time $T$ is a function of all parameters of the problem: $T\left(F,z,\delta,\rho,A^f,k_0,A^i\right)$. It is increasing in $F$, $z$, $\delta$, and $\rho$ and decreasing in $A^f$ and the initial capital stock, $k_0$. It is decreasing in $A^i$ for small values of $A^i$, then increasing.

Increases in the formalization fee $F$ will make the necessary capital accumulation take longer time. An increased cost of investing $z$ slows down the growth of the capital stock. Preformalization investments also decrease unambiguously in the depreciation rate $\delta$ and the discount rate $\rho$. In addition, an increase in $\rho$ makes firms want to postpone formalization (the RHS in 11 increases), which makes $T$ increase further. An increase in $A^f$ strengthens the incentive to invest (LHS of 11). In addition, it decreases the level of capital $k^F$ at which formalization becomes advantageous (RHS of 11). Both effects speed up formalization. The initial capital stock adds to the capital stock obtained by investing, and $T$ is therefore smaller the higher is $k_0$.

With respect to the informal productivity level $A^i$, there are two effects: an *investment effect* and a *threshold effect*. An increase in $A^i$ means more investment and capital accumulation (LHS in 11) but also that formalization becomes less advantageous (RHS in 11). For small values of $A^i$ (in comparison to $A^f$), the investment effect dominates and formalization becomes easier ($\bar{F} - F$ increases, proposition 1) and faster ($T$ decreases, proposition 2). For large values of $A^i$, the threshold effect instead dominates.

The response in $T$ to changes in the parameter values implies that there are two effects on the investment path when a parameter changes. Consider an increase in $A^f$. This produces a *direct effect* by which $i_t^{\text{formalization}}$ in (7) increases, for a given $T$. In addition, there is an *indirect effect* through a smaller formalization time $T$, which further increases investment at any moment in time. In figure 1, these direct and indirect effects could be depicted as a formalization (pre-$T$) investment path at a higher

level and with a higher slope at each point in time, a shift to the left in $T$, and a shift upwards in the formal (post-$T$) investment level. This unambiguous multiplicative effect is also present (but goes in the other direction) for changes in $z$, $\delta$, and $\rho$. Before further analyzing these results, heterogeneity between firms is introduced in section 4.6 and a microfoundation for the informal productivity parameter $A^i$, connected to a penalty policy vis-a-vis informal firms, is provided in section 4.7.

*4.6. Introducing firm heterogeneity*

The discussion so far has concerned one firm. To allow a discussion in section 5 about firms that become formal versus those that do not, an assumption about firm heterogeneity is introduced. Specifically, let firms be indexed by $j$ and assume there is a firm-specific "ability" parameter $\theta_j$ that multiplies two baseline productivity parameters, $A^I$ and $A^F$. For the sake of simplicity, let $\theta_j$ be uniformly distributed on the unit interval, $0 < \theta_j \leq 1$. The baseline parameters $A^I$ and $A^F$ can be interpreted as the maximum productivities of the informal and formal sectors, respectively. A number of reasons why these may differ were outlined in the introduction. Through its ability parameter, each firm then has its own productivity in relation to $A^I$ and $A^F$: as formal it is $A_j^f = \theta_j A^F$. The productivity of the same firm $j$ while informal, $A_j^i$, contains an additional component, discussed in the next section.

*4.7. The informal productivity $A_j^i$*

As discussed in the introduction, there are potentially many different reasons for productivity differences between informality and formality. The risk of being detected and penalized by the authorities for operating "illegally" is an often-used characterization of the informal firm environment, by informal entrepreneurs themselves (Tokman, 1992) as well as in economic models (Loayza, 1995). One reason for penalizing informal firms is that these do not pay taxes; thus, the government wants firms to formalize in order to increase tax revenue. Another rationale is that formal firms put pressure on the authorities to deal with informality, claiming that competition from non-compliers is "unfair".

An explicit story for how penalties affect informal productivity, $A_j^i$, is through the time use of informal entrepreneurs: firms spend time "hiding" from the authorities, rather than producing. This was motivated in section 2. Let the productivity of the informal firm be $\theta_j A^I$ from above if it can operate without hiding. Let $l$ be the fraction of the informal entrepreneur's unitary time endowment spent trying to avoid detection, rather than in production, let $p(l)$ be the resulting probability of not getting caught and let $x$ be the fraction of output which is taken from the informal entrepreneur if caught (the penalty/policy parameter, where $0 \leq x \leq 1$). The expected productivity when operating informally becomes $p(l)\theta_j A^I (1-l) + (1-p(l))\,\theta_j A^I (1-l)(1-x)$, which can be rewritten as[16]

$$\theta_j A^I (1-l) - x\theta_j A^I (1-l)(1-p(l)). \tag{12}$$

The first term reflects production and the second term the effect of penalties. Let the probability of not being detected be $p(l) = \sqrt{l}$. This function fulfills the natural requirements that $p(0) = 0$, $p(1) = 1$ and also $dp/dl_{l=0} = \infty$ and $d^2p/dl^2 < 0$. By solving for the optimal time allocation and detection probability from the first-order condition $-\theta_j A^I + p'(l)x\theta_j A^I (1-l) + x\theta_j A^I (1-p(l)) = 0$, we get the informal productivity parameter.[17] It is a strictly decreasing and convex function $h(x)$ of the penalty parameter $x$, where $h(0) = 1$, multiplied by $\theta_j A^I$:

$$A_j^i = \theta_j A^I h(x) \tag{13}$$

The penalty parameter thus has a negative effect on productivity. Firms can shield themselves from the worst case, by allocating time to hiding rather than to production ($h(x)$ is always larger than $1-x$).[18] Figure 2 shows the resulting informal productivity, $A_j^i = \theta_j A^I h(x)$, as a function of $x$ for an individual firm $j$. The formal productivity $A_j^f = \theta_j A^F$ is also shown. The effects of penalties on formalization are discussed in section 5.

**Figure 2 about here**

[16] The capital stock $k$ is omitted because the time allocation decision to maximize expected "per period" production is static and independent of the dynamic investment decision in (3).

[17] See appendix 2.

[18] We could use a more general function for the probability; $p(l) = l^\xi$ with $0 < \xi < 1$. The parameter $\xi$ would in a sense reflect the strength of enforcement of penalties $x$. A lower value of $\xi$ would mean that even small amounts of time used for "hiding" are very effective in avoiding detection and could be interpreted as weak enforcement.

## 5. Analysis of the model

What does the model imply in terms of formalization and investment? This section discusses a few predictions, starting out with a proposition about which firms that formalize, the investment paths, time of formalization and firm size at formalization.

### *5.1. Characteristics of firms that become formal*

From expression (11), and with productivities $A_j^i = \theta_j A^I h(x)$ and $A_j^f = \theta_j A^F$as derived in sections 4.6-4.7, and disregarding the effect of initial capital[19], we get the following proposition:

**Proposition 3**. Firms with an ability parameter $\theta_j$ above a threshold value $\theta^{\text{formalization}}$, i.e. firms in the range of $\theta^{\text{formalization}} \leq \theta_j < 1$ become formal. For such firms, the larger is $\theta_j$ the larger is investment, the faster is formalization and the smaller is the firm size at which formalization takes place.

The intuition for this proposition is straightforward: firms with high $\theta_j$ both invest more due to a higher productivity (LHS of 11 increases), and they have more to gain more from formalization (RHS of 11 decreases). In a cross section of firms, we should thus not only observe that it is high ability/productivity firms that become formal, but furthermore that their firm size at formalization is smaller and the time from firm start-up to formalization is shorter. The threshold value $\theta^{\text{formalization}}$ is derived by plugging in the full expressions for firm productivities ($A_j^i = \theta_j A^I h(x)$ and $A_j^f = \theta_j A^F$) in the formalization criterion derived in the first part of proposition 1 and solving for $\theta_j$, which gives

$$\theta^{\text{formalization}} \equiv \sqrt{\frac{A^F F z \delta (\delta + \rho)(2\delta + \rho)}{(A^F - A^I h(x))(A^I h(x)(\delta + \rho) + \delta A^F)}}. \tag{14}$$

### *5.2. Penalties*

#### *5.2.1. Policy maker*

Before analyzing the effects of penalties on formalization, a highly relevant question is: Who is this policy maker? So far, the penalty policy has been connected to a somewhat diffuse "authority".

One interpretation of the penalty parameter $x$ is that it is the government that sets (and enforces) such penalties. Then, it is assumed that the government can audit informal firms and penalize them for operating illegally. In practice, this could take place through "benevolent" tax officers, police, local authorities etc. One reason for such audits to take place may be that the government wants to increase tax revenue by making firms formal or that there is some negative externality from informal production.

An alternative view on policy, which is very different, is when there is no government in the traditional sense. Indeed, we are studying the informal sector which, by definition, consists of unregistered firms. The penalty parameter $x$ might instead be collected by "malevolent" police, corrupt bureaucrats, local mafias etc. (De Soto, 1989; Tokman, 1992). The likely aim is then not to speed up formalization, but to maximize bribe revenue from informal firms. Although I do not provide any formal analysis of such a case, there is no reason to believe that penalties would be set as in the "benevolent" case. Instead, one can hypothesize about the effects of short time horizons of "collectors" ($\Rightarrow x \uparrow$), lack of commitment to refrain from collecting more bribes ($\Rightarrow x \uparrow$), risk of being detected if collecting too much ($\Rightarrow x \downarrow$), no desire that firms should become formal and disappear from the "tax base", and so on.

In the following section, the policy maker is the government and the optimal penalty for maximizing firm formalization is derived.

#### *5.2.2. Effects from penalties on formalization*

By analyzing $\theta^{\text{formalization}}$, we can study how the government policy parameter vis-a-vis informal firms affects formalization. Whereas $\theta^{\text{formalization}}$ increases in $F$, $z$, $\delta$ and $\rho$ and decreases in $A^F$, there is an ambiguous effect with respect to $A^I h(x)$. This effect was observed in analyzing proposition 2 and is restated here:

[19] If not explicitly stated, I assume that the initial capital of firms is small, such that there is no $k_0$-dependence in whether firms formalize or not (see proposition 1).

**Proposition 4**. The effect of the penalty parameter on the threshold for formalization $\theta^{\text{formalization}}$ is U-shaped, first decreasing in $x$ for small values of $x$ and then increasing. The penalty parameter that minimizes $\theta^{\text{formalization}}$, i.e. that maximizes the amount of firms that formalize, is $x = 0$ when $A^I \leq \frac{A^F}{2}\frac{\rho}{\delta+\rho}$, in an intermediate range of $A^I$ it is given by the $x$ that solves $h(x) = \frac{A^F}{2A^I}\frac{\rho}{\delta+\rho}$ and it is $x = 1$ when $A^I \geq \frac{3\sqrt{3}A^F}{4}\frac{\rho}{\delta+\rho}$.

The policy maker can affect the incentive to formalize through the penalty on informal production. For small values of $x$, the threshold effect will dominate – formalization becomes more attractive when penalties are increased. This is seen in (11'), where the level of capital at which the firm optimally formalizes (the RHS) goes down. For large penalties, it is instead the case that the investment effect will dominate – firms will accumulate less capital and will therefore not be able to become formal. This is the LHS of 11'. The penalty that maximizes the amount of firms that formalize is derived through the necessary condition for a minimum on $\theta^{\text{formalization}}$, i.e. $\partial\theta^{\text{formalization}}/\partial x = 0$. This condition gives $h(x) = \frac{A^F}{2A^I}\frac{\rho}{\delta+\rho}$ which, in turn, must lie between $h(0) = 1$ and $h(1) = \frac{2}{3\sqrt{3}}$. This gives proposition 4.

A restatement of proposition 4 is that a policy designed to incentivize firms to become formal should be conducted with a "carrot and stick" approach: neither too mild nor too tough. The accumulation- and threshold effects will be balanced and the amount of firms that become formal is maximized.

As an illustration, assume that $A^F = 1$, $A^I = \frac{A^F}{2}$ and that $\delta = \rho$. Since $\frac{1}{4} < A^I < \frac{3\sqrt{3}}{8}$, we have an interior solution and we get $h(x) = \frac{1}{2}$. Using the $h(x)$-function (appendix 2), we get that $x^* \approx 0.68$ maximizes the amount of formalization. The graph below on the shape of $\theta^{\text{formalization}}$ shows that, for this particular illustration, lower penalties will result in much less formalization, whereas higher penalties do not affect the degree of formalization to the same extent.

**Figure 3 about here**

*5.3. The formal sector productivity $A^F$*

Although we have neither made explicit the formal sector productivity parameter $A^F$, nor have specified it as a policy parameter, it is worth pointing out that increases in $A^F$ have two effects. First, investments increase (investment effect, LHS of 11). Second, the firm size at which formalization becomes beneficial goes down (threshold effect, RHS of 11). Obviously, decreasing $A^F$ has the opposite effect.

The models by Loayza (1995) and Garcia Penalosa and Turnovsky (2005) include taxes and public goods as determinants of formal sector productivity. Higher taxes and less efficient public goods provisioning in the formal sector both act to increase informality. In the present paper, we can consider these policy parameters as potential determinants of $A^F$. The preceding paragraph then clarifies two channels through which investment in the informal sector and formalization is discouraged by higher taxation and less efficient public goods provisioning in the formal sector.

*5.4. The aggregate of firms*

What does the long-run firm size distribution predicted by the model look like? Does it resemble actual firm size distributions in developing economies?

In the long run, informal firms converge to a (firm-specific) size $k_{j,\infty}^{\text{informal}} = \frac{\theta_j A^I h(x)}{z\delta(\delta+\rho)}$ and formal firms to $k_{j,\infty}^{\text{formal}} = \frac{\theta_j A^F}{z\delta(\delta+\rho)}$. The firm with an ability parameter marginally lower than $\theta^{\text{formalization}}$ thus reaches a much smaller size than had the ability parameter been somewhat larger. The size of the firm size gap, i.e. $\frac{\theta^{\text{formalization}} A^F}{z\delta(\delta+\rho)} - \frac{\theta^{\text{formalization}} A^I h(x)}{z\delta(\delta+\rho)}$, is increasing in $F$, $\rho$, $A^F$ and decreasing in $A^I h(x)$, $z$ and $\delta$. Together with the fact that $\theta^{\text{formalization}}$ increases in $F$ and that $h(x)$ decreases in $x$, we can state the following proposition:

**Proposition 5**. In the long run, the model displays a low-end range, $0 < \theta_j \leq \theta^{\text{formalization}}$ of small informal firms and a high-end range, $\theta^{\text{formalization}} \leq \theta_j \leq 1$, of large formal firms. There is a "missing middle" in firm sizes, and the size of the gap is increasing in formalization costs $F$ and penalties $x$.

Tybout (2000) documents firm size distributions for a number of developing economies and finds evidence of a "dual structure", with a large proportion of very small firms, a "missing middle" and then a few large firms. This contrasts with typical high-income countries. The author further argues that "small producers frequently operate partly or wholly outside the realm of government regulation" (Tybout 2000, page 15), discussing costs of dealing with the government as one explanation for the observed pattern. The present model shows how the profit maximization behavior of firms, with large costs of entry into formality, can generate such a "missing middle". It also delivers predictions about the size of this firm size gap. Figure 4 shows the long-run firm sizes as predicted by the model.

**Figure 4 about here**

*5.5. The dependence on initial capital, $k_0$*

Heterogeneity between firms was introduced along an ability/productivity dimension, resulting in predictions about which firms that formalize. The focus of the poverty trap literature is instead on studying how the initial wealth distribution matters for the future wealth distribution. Translated into the present paper, the question is if differences in initial capital $k_0$ can explain differences in long-run firm sizes[20]. At face value, the answer to this question should be no: there are no explicit credit constraints in the model; therefore equally productive firms should converge to the same steady state in the long run (Banerjee, 2001; McKenzie and Woodruff, 2006).

Proposition 1 states that initial capital does play a role, however. Whenever $k_0$ is larger than $k_\infty^{\text{informal}}$, the maximum level of capital that can optimally be accumulated is larger than $k_\infty^{\text{formalization}} = \frac{A^i(\delta+\rho)+\delta A^f}{z\delta(\delta+\rho)(2\delta+\rho)}$. The maximum level of capital is also increasing in $k_0$. These statements were proven in appendix 1.

A high initial capital stock gives a firm an "initial cost advantage" in reaching a certain size. That is, it is (initially) less costly for a firm starting with a high $k_0$ to reach a certain (larger) capital level than if the firm had started with a low $k_0$. This effect vanishes over time because the initial capital depreciates at a rate $\delta$. The way in which a firm starting with high $k_0$ takes advantage of the effect is to invest heavily in the beginning and then formalize early (at capital level $k^F$ from 10). Early formalization makes the additional investment worthwhile because formalization profits come closer in time. A firm starting with a lower $k_0$ has no possibility of taking advantage of the effect of initial capital since reaching $k^F$ would mean even higher initial investment losses and/or later formalization, thus implying a lower net present value of formality.

An alternative explanation to the fact that the cost advantage due to a high $k_0$ is only initial is the following: A firm that is informal forever converges to a capital stock $k_\infty^{\text{informal}}$ at which investments just compensate depreciated capital. For a firm that formalizes at some time $T$ far into the future, it is costly to deviate to any large extent from $k_\infty^{\text{informal}}$ prior to formalization. Therefore, for a $T$ that is large, investments will mimic the "ever-informal" case where depreciated capital is just replaced, as long as $T$ is not close in time. This, in turn, means that the effect of the initial capital stock will vanish prior to $T$ and, as a result, not affect the decision whether to formalize or not.

The result that $k_0$ is of importance resembles the poverty trap literature. A certain formalization level $k^F$ that would be prohibitive for firms with $k_0$ smaller than $k_\infty^{\text{informal}}$ will still allow for a range of firms with sufficiently high initial capital to formalize and converge to $k_\infty^{\text{formal}}$.

## 6. Extensions

This section discusses some modifications to the model in section 4. I abstract from firm heterogeneity in the presentation below to save on notation.

*6.1. The effect of borrowing and savings constraints*

So far, nothing hinders the firm from making losses while informal. Losses can come from three sources: early in the process due to investment costs exceeding production revenue, due to higher investment costs in anticipation of formalization, as well as in the instant when $F$ is paid. In the formulation in (3), the only concern is that the net present value of revenue exceeds the net present value of costs.

[20] In this section, I disregard the ability differences and assume all firms to be equal except for differences in $k_0$.

It is beyond the scope of this paper to review the literature on credit.[21] Instead, the introduction of a no-borrowing constraint in this model is, somewhat innocently, motivated by the observation that most informal firms do not use any credit at all. In the Brazilian representative sample of small informal firms, 6% of the firms had used credit during the three months prior to the study and 17% of the firms had any debt outstanding whatsoever (SEBRAE, 2005). These numbers seem to be quite typical. In a study of six African countries, 2% of the micro firms (1-5 employees) and 7% of the small firms (6-25 employees) had received a loan in the year prior to the study. 16% of each group had debt in the informal finance sector (Bigsten et al., 2003).[22] Levy (1993) reports small percentages of Tanzanian and Sri Lankan micro firms (1-5 employees) with formal credit. In two recent empirical papers estimating returns to capital, McKenzie and Woodruff (2006) and de Mel, McKenzie and Woodruff (2008b) find evidence on binding credit constraints for small informal firms in Mexico and Sri Lanka. In addition to borrowing constraints, savings constraints may be important in developing countries (Duflo and Banerjee, 2007; Dupas and Robinson, 2009). In the model presented here, this would mean that informal firms cannot save from current profits to pay the formalization fee. The impact of such a constraint is discussed below.

*6.1.1. No borrowing to finance investment*

First, assume that the firm cannot borrow to finance investment. Because the firm would like to invest at least an amount $i^{\text{informal}}$ from the start, the no-borrowing constraint will clearly be binding for firms that start with a small initial capital stock (imagine $k_0$ close to zero, this gives a profit flow in the unconstrained case of $A^i k_0 - \frac{z}{2}\left(i^{\text{informal}}\right)^2 < 0$, and hence a need to borrow). Formally, the following constraint should be added to the problem formulation in (3):

$$A^i k_t - \frac{z}{2} i_t^2 \geq 0 \tag{15}$$

Adding this constraint will not affect the formalization decision. Because investment in the capital stock is perfectly divisible and the return on small investment amounts is very high, it will be optimal for the firm to forsake small current profits against future higher profits generated by a larger capital stock. The firm will invest as much as possible, i.e. $i_t = \sqrt{2A^i k_t / z}$, until the constraint in (15) is no longer binding. The firm "bootstraps" its way out of the borrowing constraint. The time it takes depends on $k_0$, but the long-run capital stock and the formalization decision will not be affected.

*6.1.2. Limited borrowing to finance the formalization cost F*

The introduction of a constraint with respect to the financing of $F$ is more complicated.[23] A shortcut is as follows: Assume that a fraction $\gamma$ can be financed as before, i.e. by the firm's own financing. A fraction $(1-\gamma)$ of $F$ can be financed by borrowing, at the instant of formalization $T$, with interest rate $r$. This results in a "per-period" interest payment of $(1-\gamma) rF$. Assuming that the debt is rolled over indefinitely, the only change to the problem is in the total profit expression in (3), which becomes

$$\int_0^T \left(A^i k_t - \frac{z}{2} i_t^2\right) e^{-\rho t} dt + \int_T^\infty \left(A^f k_t - \frac{z}{2} i_t^2 - (1-\gamma) rF\right) e^{-\rho t} dt - \gamma F e^{-\rho T}.$$

The solution is only affected through condition (9) and we get $k^F = \dfrac{\gamma\rho F + (1-\gamma) rF}{A^f - A^i}$. If the firm borrows at a high interest rate $r$ ($> \rho$), formalization will become more difficult. The smaller is $\gamma$, the

[21] Banerjee (2001) reviews a number of "stylized facts" about credit markets in developing countries.

[22] As pointed out by the authors, not having credit does not mean that firms do not want it and 64% of the micro firms had either been denied credit or say they would be denied if they applied. It should also be pointed out that several studies report trade credit to be an important source of financing for small firms.

[23] The model solved in this paper assumes that there is no other asset than the firm's own capital stock in which to invest: no financial saving can be accumulated. Given the evidence on little access to savings devices in the informal sector, this assumption is not implausible. Introducing a no-borrowing constraint to finance $F$ then implies that firms would have to save "in the mattress", i.e. at zero % interest rate, in anticipation of formalization. Introducing a savings control variable ($s_t$), we get that with zero interest, total savings before formalization should at least equal $F$, i.e. $\int_0^T s dt \geq F$. Solving the model with this constraint turns out to be very complicated. However, it is likely that the period leading up to formalization would imply a trade-off between investment and "saving in the mattress", implying less capital accumulated and, as result, a smaller possibility for formalization.

less likely is formalization. The entire analysis in section 5 remains unchanged, and we can consider the credit constraint as being represented by a larger formalization cost $F$.[24]

*6.2. An alternative view on penalties*

It is often argued that informal firms run a larger risk of detection if they grow, and therefore prefer to stay small. The model can be modified to investigate this argument. Instead of firms spending time on hiding, we now modify the original informal production function. By writing informal production as $A^i k_t \left(1 - \frac{\beta}{2} k_t\right)$, rather than $A^i k_t$, we explicitly recognize that as the firm grows, a larger fraction of output will be captured (we think of penalties $x$ as incorporated into the $\beta$-term). Although it seems likely that informal firms will now grow (even) less, the incentive to formalize is also stronger than before. What does this modification to the problem in (3), stated in appendix 3, yield? Parametrizing and comparing the solution to the model in section 4 gives at hand that for small values of $A^i$ (in comparison to $A^f$), the investment effect is dominating. That is, informal firms now simply cannot grow and will formalize to a lesser extent. However, for values of $A^i$ close to $A^f$, the opposite becomes true. If informal firms are relatively productive in the original problem, the threshold effect dominates (proposition 2) and firms have little to gain from becoming formal. This effect is reversed with the new specification. That is, growth implies higher penalties and that the incentive to become formal is strengthened. For large values of $A^i$, the firm is then able to escape such penalties by formalizing.

## 7. Discussion

This paper sets up a dynamic model of profit maximizing informal firms to study investment, growth and possible entry into formality, in the face of a large formalization cost that has to be paid at one instant in time. The basic dynamic trade-off is that, on the one hand, there is an incentive to invest and grow to be able to reap the formalization benefits. On the other hand, this may require too much investment at the early stages and prove too costly, and the firm may therefore choose to remain informal.

The model generates a number of predictions. When aggregating over firms that differ along an ability dimension, the long-run firm size distribution exhibits a range of small firms and a range of larger firms but also a "missing middle", much in line with actual firm size distributions observed in developing countries (Bigsten et al. 2004, Tybout, 2000). The model is also broadly consistent with recent empirical evidence from McKenzie and Woodruff (2006). Using a representative small-firm sample from Mexico, these authors find no evidence of barriers to growth for the smallest entrepreneurial activities. However, these authors do find some support for non-convexities which are at place and act as barriers to growth for larger firms. In line with their evidence, the present model generates a firm distribution where there is a range of small firms, growing up to a certain size but remaining informal and unable to grow further due to the formalization cost. Some firms however, by investing a lot at early stages, manage to surpass this barrier to growth and can then grow further due to a higher productivity once formal. In addition to predictions related to long-run firm sizes, the model also predicts that the firm size at formalization, as well as the timing of formalization, is firm-specific. In particular, firms run by more able entrepreneurs formalize at smaller firm sizes and earlier than do firms run by less able entrepreneurs.

With respect to policy, the paper offers two views. If the policy maker is a benevolent government, a policy designed to incentivize firms to become formal should be conducted with a "carrot and stick" approach: neither too mild nor too tough. Such a policy will make an accumulation effect and a threshold effect balance and maximize the amount of firms that become formal. The second interpretation of policy is one in which the government is absent and where the policy maker is rather "malevolent" police, corrupt bureaucrats, local mafias etc. With the latter view on policy, growth and possible formalization seem less likely to occur.[25]

[24] The representation above is admittedly a simplified way of introducing a credit constraint. In relation to the literature on poverty traps, one might imagine that the interest rate $r$ at which the firm can borrow depends on initial capital/initial establishments, such that firms that started off at higher capital stocks represent less of a moral hazard risk to lenders. Such a specification would mean that firms with a larger initial capital stock face lower effective formalization costs, and act as a source of a poverty trap. That is, the combination of the non-convexity ($F$) and higher interest rates for firms with less initial capital could, in itself, generate a long-run distribution of capital that depends on $k_0$.

[25] An interesting theoretical result in this paper is that an "informality trap" can result in a model with only a non-convexity but without a credit constraint. This contrasts with standard neoclassical theory, where non-convexities alone should not affect long-run allocations. Economic agents could simply borrow to overcome such hurdles (Banerjee, 2001; McKenzie and Woodruff, 2006). In the present model, it is the combination of adjustment costs of investment and the formalization cost that makes initial capital matter for long-run capital distributions.

| Region | 1<br>Number of procedures | 2<br>Time (days) | 3<br>Cost to start firm / (GNI/capita) |
|---|---|---|---|
| East Asia & Pacific | 8.6 | 44.2 | 32.3 |
| Eastern Europe & Central Asia | 7.7 | 22.6 | 8.6 |
| Latin America & Caribbean | 9.7 | 64.5 | 39.1 |
| Middle East & North Africa | 8.4 | 23.5 | 41.0 |
| South Asia | 7.4 | 32.5 | 31.9 |
| Sub-Saharan Africa | 10.2 | 47.8 | 111.2 |
| | | | |
| OECD | 5.8 | 13.4 | 4.9 |
| United States | 6 | 6 | 0.7 |

**Table 1.** Number of procedures, duration and cost to register a business in different parts of the world. Source: World Bank, 2009a.

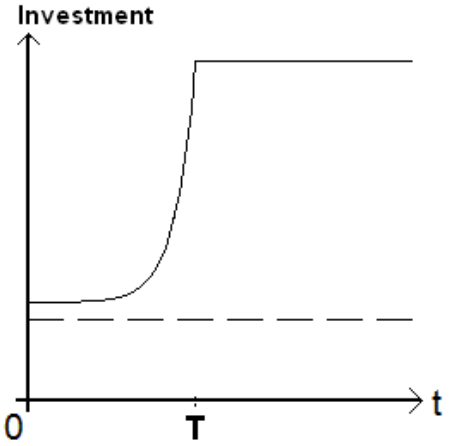


**Figure 1.** Investment paths.

| Country | 1<br>Number of procedures | 2<br>Time (days) | 3<br>Cost to start a firm (USD) | 4<br>Official monthly GNI (USD) | 5<br>Informal monthly GNP (USD) | 6<br>Start up cost/ (Informal monthly GNP) | 7<br>Total cost/ (Informal monthly GNP) |
|---|---|---|---|---|---|---|---|
| Argentina | 14 | 31 | 702 | 504 | 128 | 5,5 | 6,1 |
| Bolivia | 15 | 50 | 1891 | 105 | 70 | 26,8 | 27,6 |
| Brazil | 17 | 152 | 585 | 493 | 196 | 3,0 | 4,7 |
| Chile | 9 | 27 | 818 | 696 | 138 | 5,9 | 6,4 |
| Colombia | 13 | 44 | 644 | 271 | 106 | 6,1 | 6,8 |
| Costa Rica | 12 | 77 | 1279 | 463 | 121 | 10,5 | 11,5 |
| Dom. Rep. | 9 | 72 | 1072 | 296 | 95 | 11,3 | 12,1 |
| Ecuador | 14 | 65 | 979 | 257 | 88 | 11,1 | 12,0 |
| Guatemala | 13 | 30 | 1271 | 203 | 105 | 12,1 | 12,7 |
| Honduras | 13 | 44 | 970 | 133 | 66 | 14,7 | 15,4 |
| Mexico | 8 | 27 | 1184 | 695 | 209 | 5,7 | 6,1 |
| Nicaragua | 6 | 39 | 1290 | 82 | 37 | 34,9 | 35,4 |
| Panama | 7 | 19 | 1317 | 459 | 294 | 4,5 | 4,8 |
| Peru | 10 | 72 | 1121 | 288 | 172 | 6,5 | 7,4 |
| Uruguay | 10 | 43 | 2820 | 532 | 272 | 10,4 | 11,0 |
| Venezuela | 16 | 141 | 1859 | 610 | 205 | 9,1 | 10,6 |
| | | | | | | | |
| **Average LA** | **12** | **58** | **1238** | **380** | **144** | **11,1** | **11,9** |
| | | | | | | | |
| **United States** | **6** | **6** | **368** | **3837** | | | |

**Table 2.** Number of procedures, duration and cost to start a firm in Latin America (columns 1-3). Official and informal per capita income figures (columns 4-5). Ratio between the cost to start a firm and informal monthly GNP (column 6) and ratio between a total cost measure, incorporating time costs, and informal monthly GNP (column 7). The sources are Schneider (2002) and World Bank (2009a, 2009b).

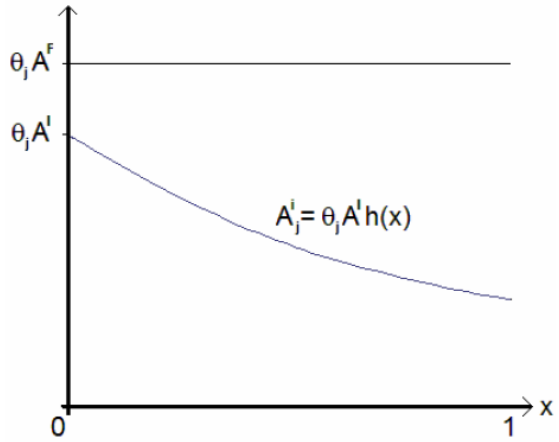


**Figure 2.** The informal and formal productivities of an individual firm *j*, as a function of penalties *x*.

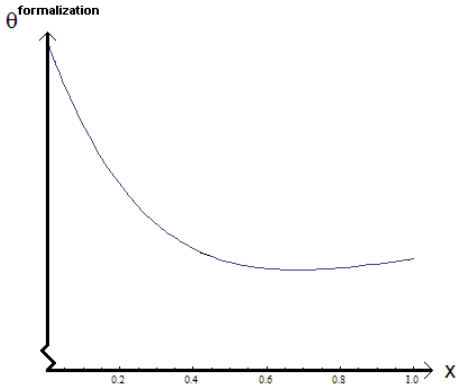


**Figure 3.** An illustration of the effects of penalties on the minimum ability threshold for formalization.

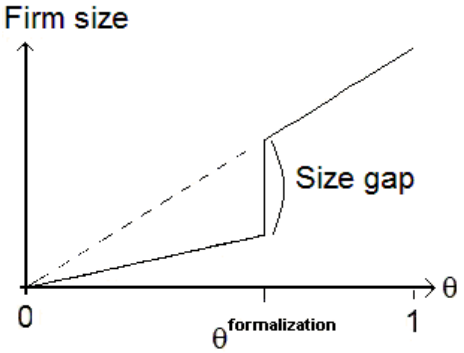


**Figure 4**. Long-term firm size distribution.

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**Appendix**

*A1: Proof of proposition 1*

Define the auxiliary capital level $k_{\infty}^{\text{formalization}} = \dfrac{A^i(\delta+\rho)+\delta A^f}{z\delta(\delta+\rho)(2\delta+\rho)}$. This is the (hypothetical) level of capital that a firm would reach at $T=\infty$ if it followed the formalization investment path forever. Using this expression, we can write the capital stock a firm reaches at the time of formalization, which is the LHS of expression (11), as

$$k_{t=T}^{\text{formalization}} = k_{\infty}^{\text{formalization}} + \left(k_0 - k_{\infty}^{\text{informal}}\right)e^{-\delta T} + \left(k_{\infty}^{\text{informal}} - k_{\infty}^{\text{formal}}\right)\frac{\delta e^{-(2\delta+\rho)T}}{2\delta+\rho}.$$

As long as $k_0 \leq k_{\infty}^{\text{informal}}$, $k_{t=T}^{\text{formalization}}$ is increasing in $T$ and converges to $k_{\infty}^{\text{formalization}}$ as $T\to\infty$. By equating $k_{\infty}^{\text{formalization}}$ with $k^F$ and solving for $F$, the formalization criterion in proposition 1 is obtained.

For $k_0 > k_{\infty}^{\text{informal}}$, the capital stock $k_{t=T}^{\text{formalization}}$ reaches a maximum of $k_{\infty}^{\text{formalization}} + \dfrac{\delta+\rho}{2\delta+\rho}\dfrac{\left(k_0 - k_{\infty}^{\text{informal}}\right)^{(2\delta+\rho)/(\delta+\rho)}}{\left(k_{\infty}^{\text{formal}} - k_{\infty}^{\text{informal}}\right)^{\delta/(\delta+\rho)}}$ at $T = \frac{1}{\delta+\rho}\ln\left(\dfrac{k_{\infty}^{\text{formal}} - k_{\infty}^{\text{informal}}}{k_0 - k_{\infty}^{\text{informal}}}\right)$.

The capital stock that can be obtained is thus larger than $k_{\infty}^{\text{formalization}}$ and depends on the initial stock of capital. As a result, the firm can face a higher formalization fee and still optimally choose to formalize. Comparing this capital stock with $k^F$ and solving for $F$, the second part of the proposition is obtained (with $G(k_0) \equiv \left(\dfrac{A^f - A^i}{\rho}\right)\dfrac{\delta+\rho}{2\delta+\rho}\dfrac{\left(k_0 - k_{\infty}^{\text{informal}}\right)^{(2\delta+\rho)/(\delta+\rho)}}{\left(k_{\infty}^{\text{formal}} - k_{\infty}^{\text{informal}}\right)^{\delta/(\delta+\rho)}}$).

*A2: Solving for the time allocated to hiding*

The first-order condition $-\theta_j A^I + p'(l)x\theta_j A^I(1-l) + x\theta_j A^I(1-p(l)) = 0$ gives the following solution:

$$l^* = \frac{2-4x+5x^2-2\sqrt{(x-1)^2(1-2x+4x^2)}}{9x^2} \qquad p^* = \sqrt{l^*}$$
$$A_j^i = \theta_j A^I h(x) \text{ where } h(x) = 1 - l^* - x(1-l^*)(1-p^*).$$

Except for $h(0)=1$ and $h(1) = \dfrac{2}{3\sqrt{3}}$, we also have that $h'(x)<0$ and $h''(x)>0$

*A3: Penalties that increase in $k_t$*

Choose $i_t$, $T$ to

$$\text{Max}\left[\int_0^T\left(A^i k_t\left(1-\frac{\beta}{2}k_t\right) - \frac{z}{2}i_t^2\right)e^{-\rho t}dt + \int_T^{\infty}\left(A^f k_t - \frac{z}{2}i_t^2\right)e^{-\rho t}dt - Fe^{-\rho T}\right] \text{ s.t. } \dot{k}_t = i_t - \delta k_t \text{ and } k(0) = k_0$$

The optimal investment path, prior to formalization, becomes:

$$i_t = \delta\frac{C_2}{C_1} + C_4(\delta+r_1)e^{r_1 t} + C_5(\delta+r_2)e^{r_2 t}$$
$$k_t = \frac{C_2}{C_1} + C_4 e^{r_1 t} + C_5 e^{r_2 t}$$

where

$$C_1 = \frac{\beta}{z} + \delta^2 + \delta\rho,\ C_2 = \frac{A_i}{z}$$
$$r_1 = \frac{\rho}{2} + \sqrt{\left(\frac{\rho}{2}\right)^2 + C_1},\ r_2 = \frac{\rho}{2} - \sqrt{\left(\frac{\rho}{2}\right)^2 + C_1}$$
$$C_3 = \left(zC_1\left((\delta+r_1)e^{r_1 T} - (\delta+r_2)e^{r_2 T}\right)\right)^{-1}$$
$$C_4 = C_3\left(C_1 F - \delta C_2 z + e^{r_2 T}(C_2 - C_1 k_0)(\delta+r_2)z\right)$$
$$C_5 = C_3\left(-C_1 F + \delta C_2 z - e^{r_1 T}(C_2 - C_1 k_0)(\delta+r_1)z\right)$$

After formalization, the expressions in (4) apply.